

AD-A168 474

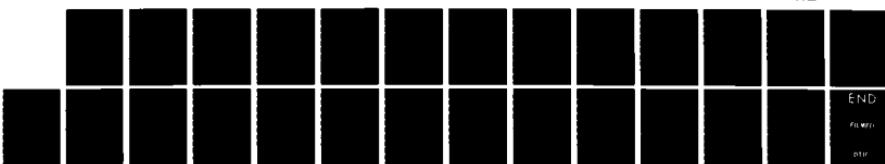
A STATISTICAL GRAVITY MODEL FOR NORTHERN TEXAS(U)  
ANALYTIC SCIENCES CORP READING MA J V WHITE 19 NOV 84  
4423-5 AFGL-TR-85-0037 F19628-83-C-0053

1/1

UNCLASSIFIED

F/G 12/1

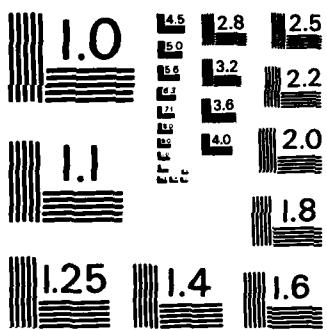
NL



END

END

01R



MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS - 1963 - A

AFGL-TR-85-0037

(12)

**A STATISTICAL GRAVITY MODEL  
FOR NORTHERN TEXAS**

**AD-A160 474**

James V. White

THE ANALYTIC SCIENCES CORPORATION  
One Jacob Way  
Reading, MA 01867

Scientific Report No. 3

19 November 1984

Approved for public release; distribution unlimited

ATIC FILE COPY

Prepared for:  
AIR FORCE GEOPHYSICS LABORATORY  
AIR FORCE SYSTEMS COMMAND  
United States Air Force  
Hanscom AFB, Massachusetts 01731

DTIC  
ELECTRONIC  
OCT 17 1985  
S A

85 10 1 124

CONTRACTOR REPORTS

This technical report has been reviewed and is approved for publication.

*Christopher Jekeli*  
CHRISTOPHER JEKELI  
All Contract Manager

*Tom Rooney*  
THOMAS P. ROONEY  
Chief, Geodesy & Gravity Branch

FOR THE COMMANDER

*Donald H. Eckhardt*  
DONALD H. ECKHARDT  
Director  
Earth Sciences Division

This report has been reviewed by the ESD Public Affairs Office (PA) and is releasable to the National Technical Information Service (NTIS).

Qualified requesters may obtain additional copies from the Defense Technical Information Center. All others should apply to the National Technical Information Service.

If your address has changed, or if you wish to be removed from the mailing list, or if the addressee is no longer employed by your organization, please notify AFGL/DAA, Hanscom AFB, MA 01731. This will assist us in maintaining a current mailing list.

## UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

AD-A160474

## REPORT DOCUMENTATION PAGE

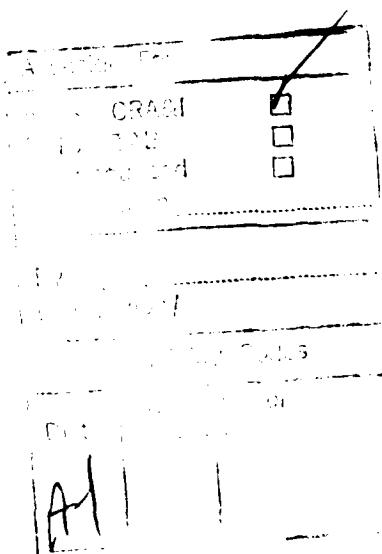
1a. REPORT SECURITY CLASSIFICATION Unclassified		1b. RESTRICTIVE MARKINGS None	
2a. SECURITY CLASSIFICATION AUTHORITY N/A		3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for Public Release Distribution Unlimited	
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE N/A			
4. PERFORMING ORGANIZATION REPORT NUMBER(S) 4423-5		5. MONITORING ORGANIZATION REPORT NUMBER(S) AFGL-TR-85-0037	
6a. NAME OF PERFORMING ORGANIZATION The Analytic Sciences Corp	6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION Air Force Geophysics Laboratory	
6c. ADDRESS (City, State and ZIP Code) One Jacob Way Reading, MA 01867		7b. ADDRESS (City, State and ZIP Code) Hanscom AFB Ma 01731	
8a. NAME OF FUNDING/SPONSORING ORGANIZATION Defense Mapping Agency	8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER F19628-83-C-0053	
8c. ADDRESS (City, State and ZIP Code)		10. SOURCE OF FUNDING NOS.	
		PROGRAM ELEMENT NO.	PROJECT NO.
		63701B	3201
		3201S46	3201DMAE
11. TITLE (Include Security Classification) A Statistical Gravity Model For Northern Texas			
12. PERSONAL AUTHOR(S) James V. White		13a. TYPE OF REPORT Scientific Report #3	
13b. TIME COVERED From May 84 to Oct 84		14. DATE OF REPORT (Yr., Mo., Day) 84 Nov 19	
15. PAGE COUNT 30			
16. SUPPLEMENTARY NOTATION None			
17. COSATI CODES	18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number) Gravity Model, Covariance function; Mass Dipoles. ←		
FIELD 08	GROUP 05	SUB. GR. .03	
19. ABSTRACT (Continue on reverse if necessary and identify by block number) A statistical gravity field model has been developed for northern Texas, where the Gravity Gradiometer Survey System (GGSS) will be tested. The model is described in this report using both round-earth and flat-earth Attenuated White Noise (AWN) formulations. The model is fitted to all worldwide and local gravity-field information that is applicable to the GGSS test site. In addition, a discrete approximation to the AWN model is described, which can be used to compute simulated GGSS survey data and surface truth data for the test area. It is expected that the AWN model and the derived simulated data will be useful for optimizing, testing, and validating GGSS data processing algorithms and software.			
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED <input checked="" type="checkbox"/> SAME AS RPT. <input type="checkbox"/> DTIC USERS <input type="checkbox"/>		21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED	
22a. NAME OF RESPONSIBLE INDIVIDUAL DONNA L. WARNER		22b. TELEPHONE NUMBER (Include Area Code) 861-3486	22c. OFFICE SYMBOL LWG

## FOREWORD

A statistical gravity field model has been developed for northern Texas, where the Gravity Gradiometer Survey System (GGSS) will be tested. The model is described in this report using both round-earth and flat-earth Attenuated White Noise (AWN) formulations. The model is fitted to all worldwide and local gravity-field information that is applicable to the GGSS test site. In addition, a discrete approximation to the AWN model is described, which can be used to compute simulated GGSS survey data and surface truth data for the test area. It is expected that the AWN model and the derived simulated data will be useful for optimizing, testing, and validating GGSS data processing algorithms and software.

## TABLE OF CONTENTS

	<u>Page No.</u>
FOREWORD	iii
List of Figures	vi
List of Tables	vi
1. INTRODUCTION	1-1
2. TECHNICAL DISCUSSION	2-1
2.1 Mass Density Doublets	2-1
2.2 AWN Gravity Models	2-3
2.3 Discrete Approximation to AWN Model	2-9
2.3.1 Parameter Values	2-10
2.3.2 Computational Formulas	2-11
2.3.3 Computational Requirements	2-14
3. SUMMARY	3-1
4. ACKNOWLEDGEMENTS	4-1
REFERENCES	R-1



### LIST OF FIGURES

<u>Figure No.</u>		<u>Page No.</u>
2.1-1	Mass Singularity and Cartesian Coordinate System	2-2
2.2-1	Undulation Degree Variances of Gem 10B, Rapp 180, and Round-Earth AWN Model	2-7
2.2-2	Undulation Degree Variances of Rapp 180 and Round-Earth AWN Model	2-7
2.2-3	Vertical Gravity Disturbance Spectra of Round-Earth and Flat-Earth AWN Models	2-8
2.2-4	Along-Track Disturbance Spectra of Flat-Earth AWN Model and Texas Data Model	2-9
2.3-1	Data Window Centered on Point (x,y)	2-13

### LIST OF TABLES

<u>Table No.</u>		
2.2-1	Round-Earth AWN Model Parameters	2-4
2.2-2	Flat-Earth AWN Model Parameters	2-4

## 1.

INTRODUCTION

This report was prepared in response to the need for a statistical gravity model that is tuned to the north Texas area where the Gravity Gradiometer Survey System (GGSS) will be tested. New round-earth and flat-earth Attenuated White Noise (AWN) gravity models (Ref. 1) have been developed and are described for the GGSS test area. These models are consistent with world-wide gravity models and with surface gravity data from the test site provided by the Defense Mapping Agency (DMA). In addition, an approach is described for generating simulated GGSS airborne survey and surface truth data for the Texas test area. This approach is based on an accurate discrete approximation to the continuous flat-earth AWN gravity model for northern Texas. The simulated data, while different from actual gravity values in the test area, nevertheless are consistent with a statistical description of the earth's true gravity field.

The discrete approximation to a flat-earth AWN model consists of a set of mass density doublets, which are uniformly spaced on horizontal rectangular grids located beneath the earth's surface. (A mass density doublet is the mathematical limit of a point-mass dipole for which the separation of the positive and negative masses approaches zero while the mass values approach infinity to keep the dipole moment finite.) Each grid of doublets in the discrete approximation corresponds to one white-noise layer of the AWN model. Each grid is at the same depth as the corresponding layer in the AWN model. The spacing of the doublets is uniform in each grid and determined by the depth. The magnitudes of the doublets are independent random variables with variances that depend on the parameters of the AWN model.

## 2.

TECHNICAL DISCUSSION

## 2.1 MASS DENSITY DOUBLETS

In this section, formulas are derived for the gravity potential, disturbance vector, and gradient tensor of a mass density doublet. These formulas are the basis for computing realizations of AWN model gravity fields.

As illustrated in Fig. 2.1-1, the gravitational potential  $T_{pm}(x,y,z)$  at point  $(x,y,z)$ , caused by a point mass singularity located at depth  $d_i$  beneath the earth's surface at point  $(x_j, y_k, -d_i)$ , is given by the formula

$$T_{pm}^{ijk}(x,y,z) = \frac{a_{ijk}'}{r_{ijk}} \quad (2.1-1)$$

where

$a_{ijk}'$  = Amplitude parameter

$r_{ijk}$  = Distance between point mass and observation point  $(x,y,z)$

$$r_{ijk} = \sqrt{(x-x_j)^2 + (y-y_k)^2 + (z+d_i)^2} \quad (2.1-2)$$

The potential  $T(x,y,z)$ , caused by a mass density doublet that is aligned with the  $z$  axis and located at  $(x_j, y_k, -d_i)$ , is proportional to the derivative of the potential of the point mass:

$$T^{ijk}(x,y,z) = c \frac{\partial}{\partial z} T_{pm}^{ijk}(x,y,z) \quad (2.1-3)$$

In Eq. 2.1-3, the constant  $c = 1$  km when  $z$  is measured in kilometers.

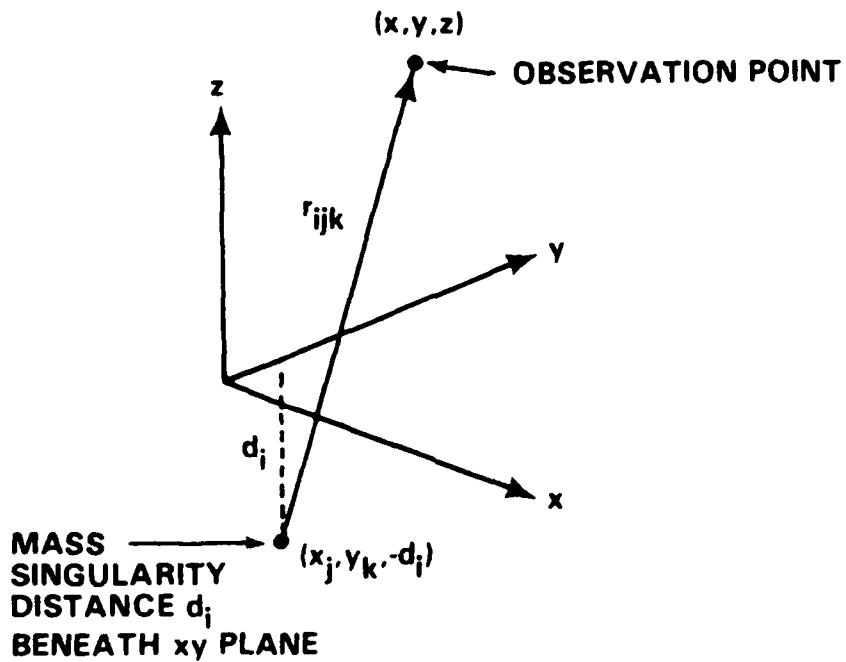


Figure 2.1-1 Mass Singularity and Cartesian Coordinate System

$$T^{ijk}(x, y, z) = -\frac{a_{ijk} (z+d_i)}{r_{ijk}^3} \quad (2.1-4)$$

In Eq. 2.1-4, the parameter  $a_{ijk}$  has units of  $\text{mGal} \cdot \text{km}^3$ .

The three components of the gravitational disturbance caused by the doublet are obtained from Eq. 2.1-4 by differentiating with respect to  $x$ ,  $y$ , and  $z$ :

$$T_x^{ijk}(x, y, z) = \frac{3 a_{ijk} (x-x_j)(z+d_i)}{r_{ijk}^5} \quad (2.1-5)$$

$$T_y^{ijk}(x, y, z) = \frac{3 a_{ijk} (y-y_k)(z+d_i)}{r_{ijk}^5} \quad (2.1-6)$$

$$T_z^{ijk}(x, y, z) = \frac{a_{ijk} [3(z+d_i)^2 - r_{ijk}^2]}{r_{ijk}^5} \quad (2.1-7)$$

The elements of the gradient tensor ( $T_{xx}$ ,  $T_{xy}$ , ...,  $T_{zz}$ ) are obtained from Eqs. 2.1-5 to 2.1-7 by further differentiations:

$$T_{xx}^{ijk}(x, y, z) = - \frac{3 a_{ijk} [5(x-x_j)^2 - r_{ijk}^2] (z+d_i)}{r_{ijk}^7} \quad (2.1-8)$$

$$T_{xy}^{ijk}(x, y, z) = - \frac{15 a_{ijk} (x-x_j)(y-y_k)(z+d_i)}{r_{ijk}^7} \quad (2.1-9)$$

$$T_{xz}^{ijk}(x, y, z) = - \frac{3 a_{ijk} (x-x_j) [5(z+d_i)^2 - r_{ijk}^2]}{r_{ijk}^7} \quad (2.1-10)$$

$$T_{yy}^{ijk}(x, y, z) = - \frac{3 a_{ijk} [5(y-y_k)^2 - r_{ijk}^2] (z+d_i)}{r_{ijk}^7} \quad (2.1-11)$$

$$T_{yz}^{ijk}(x, y, z) = - \frac{3 a_{ijk} (y-y_k) [5(z+d_i)^2 - r_{ijk}^2]}{r_{ijk}^7} \quad (2.1-12)$$

$$T_{zz}^{ijk}(x, y, z) = - \frac{3 a_{ijk} (z+d_i) [5(z+d_i)^2 - 3r_{ijk}^2]}{r_{ijk}^7} \quad (2.1-13)$$

## 2.2 AWN GRAVITY MODELS

Round-earth and flat-earth AWN models are given in Ref. 1 for modeling world-wide average gravity. To support the GGSS test program, new round- and flat-earth AWN models have been developed and are described in this report. They are based on global degree-variance data and on local surface gravity data from the GGSS test area in northern Texas. The

parameters of these AWN models are listed in Tables 2.2-1 and 2.2-2. (In these tables,  $D(i)$  is the depth of the  $i^{\text{th}}$  white-noise shell or layer, and  $\sigma_T(i)$  is the rms potential at the earth's surface caused by this shell).

TABLE 2.2-1  
ROUND-EARTH AWN MODEL PARAMETERS

i	$D(i)$	$\sigma_T(i)$
SHELL NO.	DEPTH (km)	rms POTENTIAL (mGal · km)
1	2.1	2.3
2	5.0	11
3	16	72
4	52	580
5	160	2300
6	840	7000
7	2000	33000

TABLE 2.2-2  
FLAT-EARTH AWN MODEL PARAMETERS

i	$D(i)$	$\sigma_T(i)$
LAYER NO.	DEPTH (km)	rms POTENTIAL (mGal · km)
1	2.1	2.3
2	5.0	11
3	16	72
4	52	580
5	161	2300
6	861	7000
7	2150	33000

The AWN models in Tables 2.2-1 and 2.2-2 were fitted to the following data:

- Gem 10B undulation degree variances through degree 36
- Rapp 180 undulation degree variances through degree 100
- Along-track surface gravity data from northern Texas provided by the Defense Mapping Agency (DMA).

The four shallowest white-noise shells of the new round-earth AWN model were fitted to the northern Texas surface gravity data provided by DMA (Ref. 2). The depths and rms potentials of these shells were selected to yield a great-circle gravity disturbance power spectrum that accurately approximates the along-track disturbance power spectrum that was estimated by TASC using the DMA surface data. (The spectrum estimate is based on a technique for state-space modeling (Ref. 3), which allows multiple tracks of data to be pooled for low-frequency spectrum estimates that are limited by the total extent of the data, rather than the lengths of individual tracks.) The parameters of the two deepest shells were selected to model the Gem 10B (Ref. 4) and Rapp 180 (Ref. 5) undulation degree-variance data. The third deepest shell was selected to fit simultaneously the along-track disturbance spectrum of the northern Texas data and the Rapp 180 undulation degree variances to degree 100.

Once the parameters of the new round-earth AWN model had been determined, the corresponding flat-earth AWN model was defined as follows. The flat-earth AWN model has one layer for each shell in the round-earth model. The rms undulation caused by each layer in the flat-earth approximation is required to be the same as the rms undulation of the corresponding shell

in the round-earth model. The depth of each layer in the flat-earth model is selected to produce the same rms gravity disturbance at the earth's surface as the corresponding shell produces in the round-earth model.

The round-earth AWN undulation degree-variance spectrum is compared with the corresponding spectra of Gem 10B and Rapp 180 in Fig. 2.2-1. The AWN model provides an accurate fit except for degrees 0, 1, and 3. The AWN model has significant variance in degrees 0 and 1, which is absent in Gen 10B and Rapp 180. As compensation for this excess variance, the AWN variance at degree 3 is less than Gen 10B and Rapp 180. The net result is that the AWN model has a total rms undulation (from all degrees) of 34.5 m, which is similar to the Gen 10B rms undulation of 30.3 m. Further attention to details of the fit at degrees 0, 1, and 3 was not pursued because the total variance of these terms is appropriate for modeling the gravity field within the finite GGSS test area (approximately 500 km by 500 km in extent).

The round-earth AWN undulation degree-variance spectrum is compared with Rapp 180 in Fig. 2.2-2. The two spectra are in general agreement except for degrees exceeding approximately 100. This departure of the AWN model from Rapp 180 is the consequence of requiring the AWN model to agree with the surface gravity disturbance data from northern Texas.

The accuracy with which the flat-earth AWN model approximates the round-earth AWN model is verified in Fig. 2.2-3, which compares the along-track gravity disturbance spectrum of the flat-earth model with the (suitably scaled) great-circle disturbance spectrum of the round-earth model. As expected, the only discernable differences between the two spectra occur at long wavelengths ( $\lambda > 2500$  km).

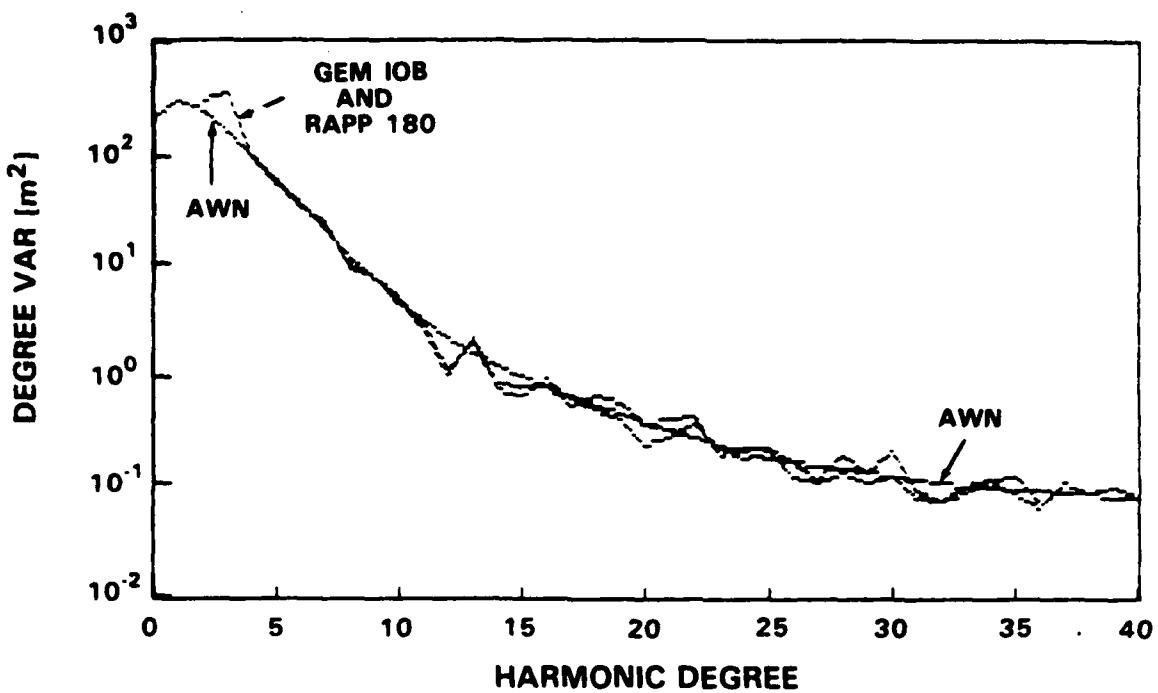


Figure 2.2-1 Undulation Degree Variances of Gem 10B, Rapp 180, and Round-Earth AWN Model

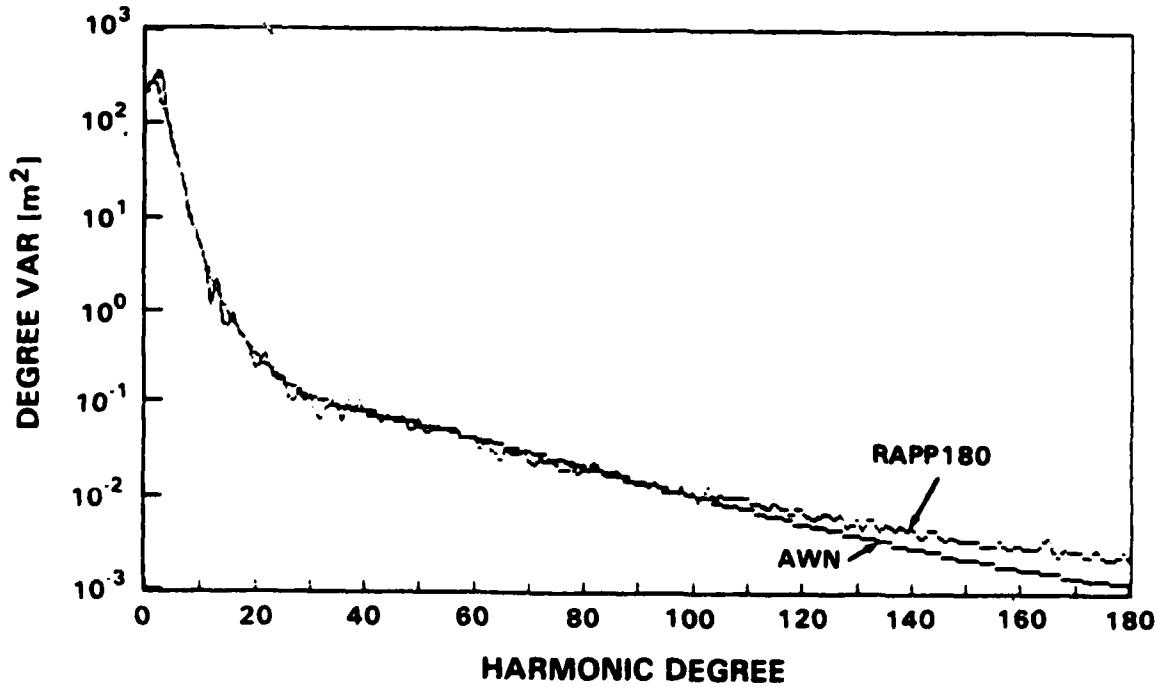


Figure 2.2-2 Undulation Degree Variances of Rapp 180 and Round-Earth AWN Model

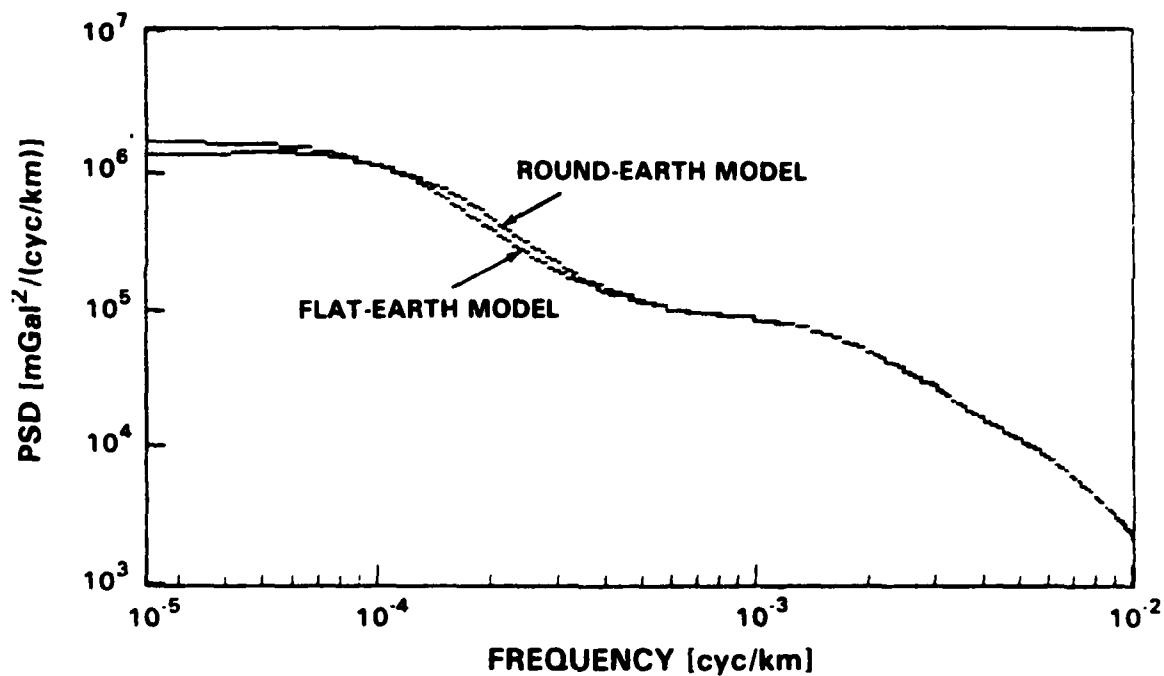


Figure 2.2-3      Vertical Gravity Disturbance Spectra of Round-Earth and Flat-Earth AWN Models

The accuracy with which the round- and flat-earth AWN models approximate the estimated along-track gravity disturbance spectrum of the DMA data from northern Texas is indicated in Fig. 2.2-4. The high-frequency limit of this comparison is determined by the sampling interval of 2.8 nm that was used in analyzing the DMA data sets.

In Fig. 2.2-4, the estimated spectrum of the Texas data is seen to flatten-off at frequencies less than approximately  $10^{-3}$  cyc/km ( $\lambda > 1000$  km). This flattening is consistent with the finite extent of the local gravity data. Below the frequency  $10^{-3}$  cyc/km (wavelengths longer than 1000 km), the AWN spectrum rises so that the round-earth AWN model is consistent with the Rapp 180 undulation degree-variance spectrum for degrees less than 100.

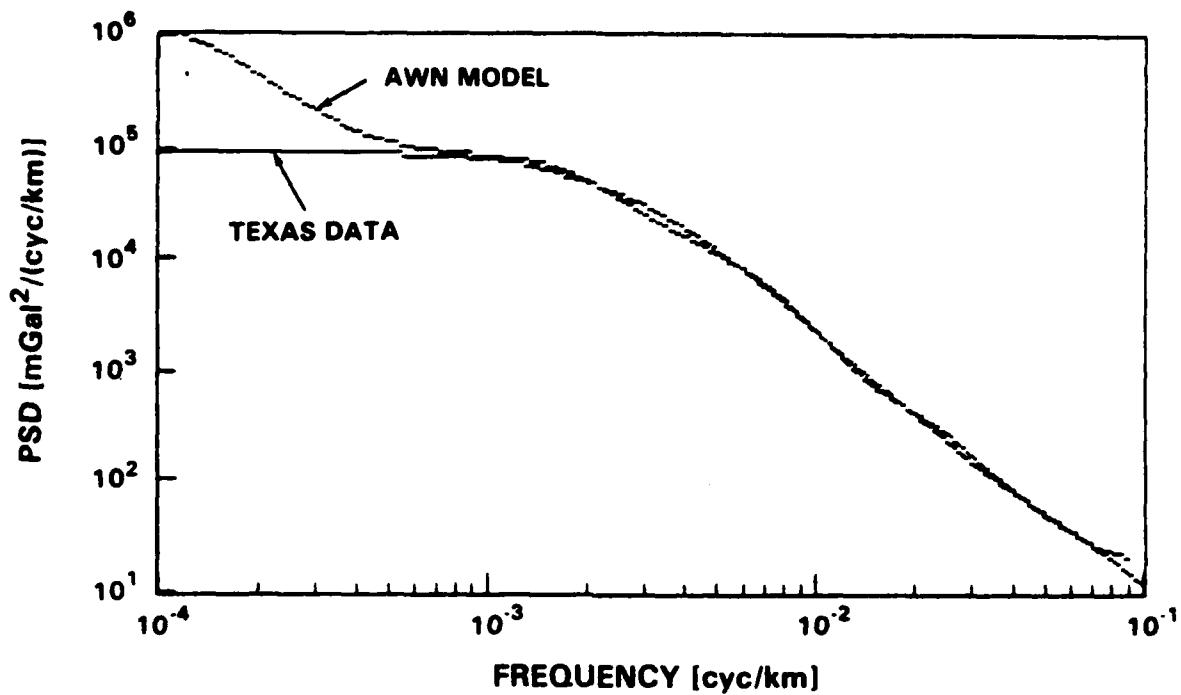


Figure 2.2-4      Along-Track Disturbance Spectra of Flat-Earth AWN Model and Texas Data Model

### 2.3 DISCRETE APPROXIMATION TO AWN MODEL

A flat-earth AWN gravity model is described by the following parameters:

- n      The number of white-noise layers
- $D(i)$       The depth (km) of the  $i^{\text{th}}$  layer, for  $i=1$  to  $n$
- $\sigma_T(i)$       The rms surface gravity potential ( $\text{mGal} \cdot \text{km}$ ) of the  $i^{\text{th}}$  layer, for  $i=1$  to  $n$ .

The discrete approximation to the AWN model is described using the additional parameters defined below:

L The linear extent (km) of the square geographic area ( $L \times L$ ) over which the gravity field is to be represented

$L(i)$  The linear extent (km) of the  $i^{\text{th}}$  grid of doublets, for  $i=1$  to  $n$

$s(i)$  The grid spacing (km) between adjacent mass density doublets in the  $i^{\text{th}}$  layer, for  $i=1$  to  $n$

$\sigma(i)$  The rms amplitude ( $\text{mGal} \cdot \text{km}^2$ ) of the doublets in the  $i^{\text{th}}$  layer, for  $i=1$  to  $n$ .

### 2.3.1 Parameter Values

Numerical experiments with discrete approximations to the flat-earth AWN model lead to the following working formulas for the doublet grid spacing  $s(i)$ , the mass density doublet rms amplitude  $\sigma(i)$ , and the linear extent  $L(i)$  of the  $i^{\text{th}}$  layer (for  $i=1$  to  $n$ , where  $n$  is the number of AWN layers):

$$s(i) = 0.4 D(i) \quad (2.3-1)$$

$$\sigma(i) = 0.3178 D(i)^2 \sigma_T(i) \quad (2.3-2)$$

$$L(i) = L + 11.5 D(i) \quad \text{for computing } T_x, T_y, T_z \\ \text{at the earth's surface} \quad (2.3-3)$$

$$L(i) = L + 7.5[D(i)+H] \quad \text{for computing } T_{xx}, T_{xy}, \dots T_{zz} \\ \text{at height } H \quad (2.3-4)$$

According to Eq. 2.3-1, the grid spacing is 40% of the layer depth. This spacing is close enough so that the variance of the gradient field  $T_{zz}$  changes from place to place within the  $L \times L$  region on the surface of the earth by less than 1% of its average value. In other words, the 40%-of-depth spacing is appropriate to provide an accurate approximation of a stationary gradient field.

The formula for rms doublet amplitude in Eq. 2.3-2 is appropriate for a grid spacing that is 40% of the depth. The coefficient, 0.3178, which was established by precise numerical covariance calculations, is close to the estimated value of 0.3192 that was obtained from an approximate analytical covariance analysis. The linear extent of the  $i^{\text{th}}$  doublet grid specified in Eq. 2.3-3 is appropriate for computing vertical gravity disturbance ( $T_z$ ) values within the  $L \times L$  region at the earth's surface with an rms accuracy that is better than 0.1 mGal. The slightly smaller grid extent given in Eq. 2.3-4 yields gravity disturbance gradient ( $T_{zz}$ ) values with an rms accuracy that is better than 0.1 E (0.01 mGal/km).

### 2.3.2 Computational Formulas

In this section, formulas are given for using an  $n$ -layer AWN model to compute gravity disturbances and gradients at an arbitrary point  $(x, y, z)$  that is either on ( $z=0$ ) or above ( $z=H$ ) the  $L \times L$  domain. The calculations consist of computing  $n$  arrays of zero-mean independent random variables with specified variances. There is one array for each layer in the AWN model. Let  $A(i)$  denote the array for the  $i^{\text{th}}$  layer, and let  $A_{jk}(i)$  denote a number in its  $j^{\text{th}}$  row and  $k^{\text{th}}$  column. The elements of  $A(i)$  are computed as follows. First, a zero-mean unit-variance random number  $v_{jk}(i)$  is computed for each grid point in the  $i^{\text{th}}$  layer. Second, these numbers are scaled by the rms doublet amplitude  $\sigma(i)$  (defined by Eq. 2.3-2) and stored in the corresponding array element:

$$A_{jk}(i) = \sigma(i) v_{jk}(i) \quad (2.3-5)$$

The expected values of the random numbers satisfy the following equations for all admissible  $i$ ,  $j$ , and  $k$ :

$$E[v_{jk}(i)] = 0 \quad (2.3-6)$$

$$E[v_{jk}(i)^2] = 1 \quad (2.3-7)$$

$$E[v_{jk}(i)v_{pq}(m)] = 0 \quad \text{for } (i,j,k) \neq (m,p,q) \quad (2.3-8)$$

The dimensions of the  $n$  arrays  $A(1) \dots A(n)$  are determined by the linear extent and grid spacing for each of the  $n$  layers. The number of elements  $[N(i)]$  in array  $A(i)$  is computed using Eqs. 2.3-1 and 2.3-4 and the  $\text{INT}(x)$  function, which returns the integer part of  $x$ :

$$N(i) = \text{INT} \left[ \frac{L + 7.5 [D(i) + H]}{0.4 D(i)} + 1 \right]^2 \quad (2.3-9)$$

As an example of the use of the discrete approximation to the AWN model, formulas are given for computing the  $T_{xy}(x,y,z)$  element of the gradient tensor at any point  $(x,y,z)$ , where  $0 \leq x \leq L$ ,  $0 \leq y \leq L$ , and  $z = H$ .

$$T_{xy}(x,y,H) = \sum_{i=1}^n \sum_{j,k \in W(i,x,y)} T_{xy}^{ijk}(x,y,H) \quad (2.3-10)$$

In Eq. 2.3-10, the limits of summation on  $j$  and  $k$  are denoted  $W(i,x,y)$ , which is square data window centered on the point  $(x,y)$  as depicted in Fig. 2.3-1. The sides of this window have length  $7.5[D(i)+H]$  when computing gradients at height  $H$ . (In contrast, the sides of the window should have length  $11.5D(i)$  when computing gravity disturbances at the earth's surface.) The window is used to keep computation time within reasonable bounds by avoiding a complete summation over the entire set of doublets for each observation point. Distant doublets, which do not contribute significantly to the gravity field at the observation point, are omitted.

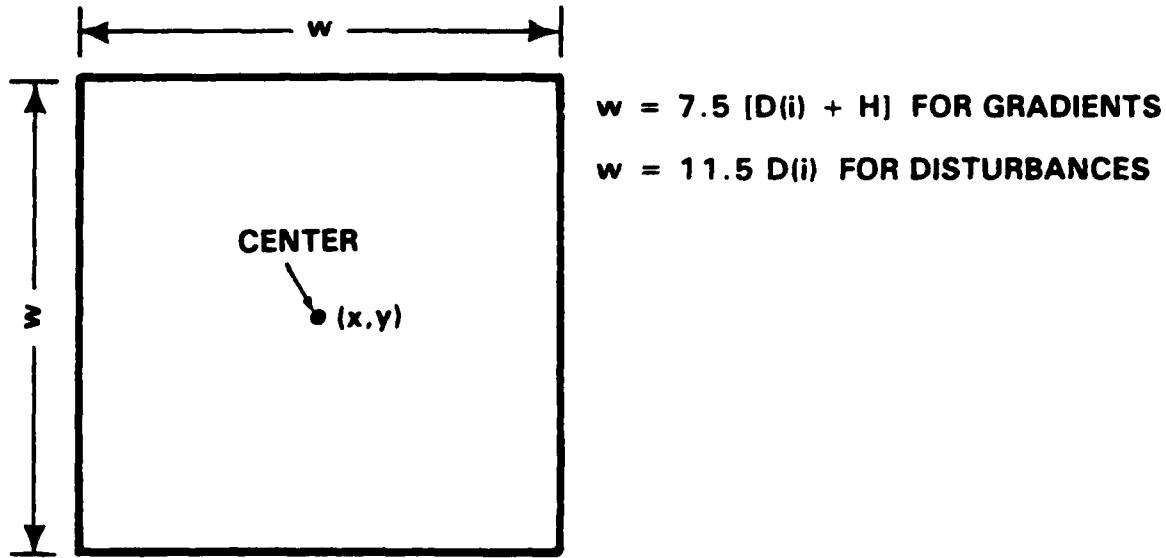


Figure 2.3-1 Data Window Centered on Point  $(x, y)$

In Eq. 2.3-10, the gradient caused by the  $jk^{\text{th}}$  doublet in the  $i^{\text{th}}$  layer is given by Eq. 2.1-9, which is expressed here using parameters  $H$  and  $D(i)$ :

$$T_{xy}^{ijk}(x, y, H) = - \frac{15 a_{ijk}(x-x_j)(y-y_k)[H+D(i)]}{r_{ijk}^7} \quad (2.3-11)$$

In Eq. 2.3-11, the radial distance is given as

$$r_{ijk} = \sqrt{(x-x_j)^2 + (y-y_k)^2 + [H+D(i)]^2} \quad (2.3-12)$$

and the doublet amplitude  $a_{ijk}$  is the appropriate random number in the array  $A(i)$ :

$$a_{ijk} = A_{jk}(i) \quad (2.3-13)$$

To compute any of the other gradient elements ( $T_{xx}$ ,  $T_{xz}$ , ...) or any of the gravity disturbance components ( $T_x$ ,  $T_y$ , or  $T_z$ ), the appropriate formula in Section 2.1 is used instead of Eq. 2.3-11.

### 2.3.3 Computational Requirements

The amount of computation that is required to compute simulated GGSS airborne survey data (e.g., gradients) for the Texas test area is estimated in this section. The test area is assumed to be a 300-km by 300-km square, and the airborne survey is assumed to have a height of 0.6 km. Therefore,  $L = 300$  km,  $H = 0.6$  km, and Eq. 2.3-4 gives the following expression for the linear extent of the  $i^{\text{th}}$  doublet layer:

$$L(i) = 300 \text{ km} + 7.5[D(i) + 0.6 \text{ km}] \quad (2.3-14)$$

The number of doublets in this layer is  $N(i)$ , where from Eq. 2.3-9

$$N(i) = \text{INT} \left[ \frac{300 \text{ km} + 7.5[D(i) + 0.6 \text{ km}]}{0.4 D(i)} + 1 \right]^2 \quad (2.3-15)$$

The number of doublets in the data window  $W(i,x,y)$  of Eq. 2.3-10 is denoted  $N_w(i)$ . For computing gradient elements aloft, this number is

$$N_w(i) = \text{INT} \left[ \frac{7.5[D(i) + 0.6 \text{ km}]}{0.4 D(i)} + 1 \right]^2 \quad (2.3-16)$$

Equation 2.3-16 gives the number of times the quantity  $T_{xy}^{ijk}(x,y,H)$  must be evaluated in Eq. 2.3-10 for each  $i$  from 1 to  $n$ . This number varies from  $N_w(1) = 625$  to  $N_w(7) = 361$ . There are six such gradient elements ( $T_{xx}$ ,  $T_{xy}$ , ...,  $T_{zz}$ ) that need to be computed, and the formulas for them (Eqs. 2.1-8 to 2.1-13)

all have about the same complexity. That is, the evaluation of each doublet contribution to the field at point  $(x, y, H)$  involves approximately 10 multiplies and divides. Therefore, the total number  $N_{md}$  of multiplies and divides to compute the six gradient elements at a single point  $(x, y, H)$  is the sum of the numbers  $N_{md}(i)$  for each layer:

$$N_{md} = \sum_{i=1}^n N_{md}(i) \quad (2.3-17)$$

$$N_{md}(i) = 6 \cdot N_w(i) \cdot 10 \quad (2.3-18)$$

The total number of multiplies and divides for computing the six gradient elements at every point  $(x, y, H)$  of interest in the airborne survey is denoted  $N_T$ . It is estimated by multiplying  $N_{md}$  in Eq. 2.3-17 by the total number of points  $N_p$  in the survey (one point every 1 km along tracks 300 km long spaced 5 km apart):

$$N_p = 301 \cdot 61 \cdot 2 = 3.67 \times 10^4 \quad (2.3-19)$$

$$N_T = N_p N_{md} \quad (2.3-20)$$

Using the AWN model parameters in Table 2.2-2, Eqs. 2.3-16 to 2.3-20 yield the following estimate for the total number of multiplies and divides:

$$N_T = 6.5 \times 10^9 \quad (2.3-21)$$

If the computer can perform  $10^6$  multiplies and divides per second then these  $6.5 \times 10^9$  mathematical operations can be performed in 1.8 hours. The arrays of random numbers  $A(1)$ ,  $A(2)$ , ...  $A(7)$  range in size from  $382 \times 382$  for  $A(1)$  to  $20 \times 20$  for  $A(7)$ . Using single precision, the total amount of computer memory required to store these arrays is approximately 730,000 bytes.

## 3.

SUMMARY

This memorandum presents a new statistical gravity field model using the Attenuated White Noise functional form. The model describes the earth's field in the north Texas area where the GGSS will be tested. The round-earth and flat-earth parameters of the model have been selected to match the statistical character of all available worldwide and local gravity-field information that is applicable to the GGSS test area.

A full description is provided of equations and parameters that can be used to realize synthetic GGSS test data and ground truth data from the model. It is anticipated that the AWN model and the derived synthetic data will be useful to the gradiometer community for the following purposes:

- Optimization of GGSS algorithms for downward continuation and estimation for the residual gravity disturbance vector
- Testing and validation of GGSS data processing software
- Analysis of GGSS data-reduction sensitivity to differences between actual field statistics and practical covariance models used for data processing
- Gaining experience in the logistics of GGSS data processing before the actual test data flow begins.

4.

ACKNOWLEDGEMENTS

Recognition is made of helpful discussions and technical inputs provided by Chris Jekeli of AFGL, Mel Schultz and John Graham of DMAAC, and Peter Ugincius of NSWC-DL, as well as others in their organizations.

## REFERENCES

1. Heller, W.G., and Jordan, S.K., "Attenuated White Noise Statistical Gravity Models," J. Geophys. Res., Vol. 84, No. B9, pp. 4680-4688 (August 1979).
2. Technical briefing presented by the Defense Mapping Agency at the GGSS Data Analysis Working Group Meeting on 3 October 1984.
3. White, J.V., "Stochastic State-Space Models from Empirical Data," Proceedings ICASSP 83, Vol. 1, IEEE Intern. Conf. Acoustics, Speech and Signal Processing, Paper No. 6.3, April 1983, pp. 243-246.
4. Lerch, F.J., Wagner, C.A., and Klosko, S.M., "Goddard Earth Models for Oceanographic Applications (GEM 10B and 10C)," Marine Geodesy, Vol. 5, pp. 145-187, 1981.
5. Rapp, R.H., "The Earth's Gravity Field to Degree and Order 180 Using Seasat Altimeter Data, Terrestrial Gravity Data, and Other Data," Report 322, Dept. Geod. Sci., Ohio State Univ., Columbus, 1981.

**END**

**FILMED**

**12-85**

**DTIC**